Becker, Ralf and Clements, Adam E. and White, Scott I. (2007) Does implied volatility provide any information beyond that captured in model-based volatility forecasts? . *Journal of Banking & Finance* 31(8):pp. 2535-2549.

Does implied volatility provide any information beyond that captured in model-based volatility forecasts?

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November 2, 2006

Abstract

This paper contributes to our understanding of the informational content of implied volatility. Here we examine whether the S&P 500 implied volatility index (VIX) contains any information relevant to future volatility beyond that available from model based volatility forecasts. It is argued that this approach differs from the traditional forecast encompassing approach used in earlier studies. The findings indicate that the VIX index does not contain any such additional information relevant for forecasting volatility.

Keywords: Implied volatility, information, volatility forecasts, volatility models, realized volatility, volatility risk premium.
JEL Classification: C12, C22, G00.
Acknowledgements The authors thank seminar participants at the University of Alabama, Queensland University of Technology, Sydney University, Melbourne University, Monash Univsersity and ESAM 2004 for useful comments. We are also grateful to Adrian Pagan, Ser-Huang Poon and two anonymous referees for their comments. The usual disclaimer applies.
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1 Introduction

As the volatility of the underlying asset price is an input into option pricing models, option traders require an expectation of this volatility before valuing options. Therefore, conditional on observed option prices (and an option pricing model) the expected volatility implied by option prices (IV) should represent a market's best prediction of an assets' future volatility (see amongst others, Jorion, 1995, Poon and Granger, 2003).

The performance of IV has often been compared to model-based forecasts (MBF) such as GARCH style models within a forecast encompassing framework. Poon and Granger (2003) provide a wide ranging survey examining the relative performance of these approaches to forecasting volatility. Specific examples of this strand of literature are Day and Lewis (1992), Lamoureux and Lastrapes (1993), and Pong, Shackleton, Taylor and Xu (2004). Overall, the majority of previous research concludes that IV yields superior forecasts of future volatility. However, in many instances, a combination of forecasts from competing approaches often including IV is preferred[1].

One drawback of this previous work is that at no stage can the chosen set of MBF be considered as a comprehensive set of forecasts because most compare IV to individual MBF. Therefore, the apparent superiority of IV may be due to the shortcomings of individual MBF used in the comparisons. However, even if one selected a comprehensive set of MBF and found that IV encompassed this set of forecasts, previous approaches are unable to determine whether IV contains information incremental to that contained in MBF. The central goal of this paper is to establish whether IV contains any incremental information that could not be obtained from the totality of information reflected in MBF, and so extend our understanding of information contained in IV and option prices.

[1] While not utilising forecast encompassing techniques, two related articles are Fleming (1998) and Blair, Poon and Taylor (2001). Fleming (1998) considers whether IV encompasses past return information often used to generate MBF. Blair, Poon and Taylor (2001) find the inclusion of IV as an exogenous variable in GARCH models to be beneficial.

The central finding demonstrated in this paper shows that IV does not contain any incremental information beyond that captured in a wide array of MBF. This result helps to explain the apparent forecast superiority of IV (see the earlier discussion of the literature and Poon and Granger, 2003) relative to individual MBF because individual forecasts are deficient relative to IV.

The paper proceeds as follows. Section 2 discusses two important data issues and the data series used for this study. Section 3 outlines the econometric volatility models which are utilised to generate volatility forecasts. The methodology used to address the research question is introduced in Section 4. Section 5 presents empirical results and section 6 provides concluding remarks.

2 Data

This study is based upon data relating to the S&P 500 Composite Index, from 2 January 1990 to 17 October 2003 (3,481 observations). To formally test the informational content of IV, estimates of both IV and future realisations of volatility were necessary.

The VIX index constructed by the Chicago Board of Options Exchange from S&P 500 index options constitutes the estimate of IV utilised in this paper. It is derived from out-of-the-money put and call options with maturities close to the target of 22 trading days. For technical details relating to the construction of the VIX index, see Chicago Board Options Exchange (CBOE, 2003). While the true process underlying option pricing is unknown, the VIX is constructed to be a general measure of the market's estimate of average S&P 500 volatility over the subsequent 22 trading days (BPT, 2001, Christensen and Prabhala, 1998 and CBOE, 2003). This index has only been available since September 2003 when the CBOE replaced an earlier version based on S&P 100 options[2]. The benefits of the current version are that it no longer relies on the Black-Scholes model, and is based on more liquid options written on the S&P500 which are easier to hedge against (CBOE, 2003).

Earlier work on the issue of informational content in IV (Day and Lewis, 1992, Fleming, 1998)

[2] This version of the VIX has been calculated retrospectively back to January 1990, the beginning of the sample considered here.

was not based on an IV index but on a rolling series of option prices from which implied volatilities were derived resulting in varying forecast horizons. The constant 22 day forecast horizon of the earlier version of the VIX is one of its major advantage acknowledged by BPT (2001). This continues to apply to the more recent version of the VIX utilised in this paper[3].

For the purposes of this study estimates of actual volatility were obtained using the realized volatility (RV) methodology outlined in Andersen, Bollerslev, Diebold and Labys (ADBL hereafter) (2001, 2003). RV estimates volatility by means of aggregating intra-day squared returns. It should be noted that the daily trading period of the S&P500 is 6.5 hours and that overnight returns were used as the first intra-day return in order to capture variation over the full calender day. ABDL (1999) suggest how to deal with practical issues relating to intra-day seasonality and sampling frequency when dealing with intra-day data. Based on this methodology, daily RV estimates were constructed using 30 minute S&P500 index returns[4]. It is widely acknowledged (see e.g. Poon and Granger, 2003) that RV is a more accurate and less noisy estimate of the unobserved volatility than squared daily returns, the measure used in a number of earlier papers (e.g. Day and Lewis, 1992, Fleming, 1998)[5].

[Insert Figure 1 here]

Figure 1 shows the VIX and daily S&P500 RV for the sample period considered. While the RV estimates behave in a similar manner to the VIX, RV reaches higher peaks than the VIX. This difference is mainly due to the VIX representing an average volatility measure for a 22 trading day period as opposed to RV which is a measure of daily volatility.

[3] The daily volatility implied by the VIX can be calculated when recognising that the VIX quote is equivalent to 100 times the annualised return standard deviation. Hence $\left(VIX/\left(100\sqrt{252}\right)\right)^2$ represents the daily volatility measure (see CBOE, 2003).

[4] Intraday S&P 500 index data were purchased from Tick Data, Inc.

[5] It should also be noted that volatility need not necessarily be measured by squared deviations from a mean, as implied by the present measure of volatility. Alternatives are the mean absolute deviation or the interquartile range (see Poon and Granger, 2003, for a brief discussion of these alternative measures). As we use commonly applied econometric models of volatility to span the space of all available historical information, this paper follows the convention to regard a measure of squared deviation as the appropriate volatility measure.

3 Models of volatility

To address the question of whether VIX reflects more information than that contained in MBF, it is necessary to specify the models upon which the forecasts are based. While the true process underlying the evolution of volatility is not known, a range of candidate models exist and are chosen so that they span the space of available model classes. Therefore, as outlined in Section 1, this builds upon previous works because we consider the informational content of VIX relative to a wide set of models.

The models chosen include models from the GARCH, Stochastic volatility (SV), and RV classes in a similar manner to Koopman, Jungbacker and Hol (2004) and BPT (2001). Forecasts generated by the MIDAS approach (Ghysels, Santa-Clara and Valkanov, 2006) are also considered.[6] In the current section, the specification of each model will be introduced along with parameter estimates based on the entire dataset. These models will then be used to recursively generate volatility forecasts in the subsequent section.

GARCH style models employed in this study are similar to those proposed by BPT (2001). The simplest model specification is the GJR (see Glosten *et al.*, 1993, Engle and Ng, 1991) process,

$$\begin{aligned} r_t &= \mu + \varepsilon_t \qquad \varepsilon_t = \sqrt{h_t} z_t \qquad z_t \sim N(0,1) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \tag{1}$$

that captures the asymmetric relationship between volatility and returns. The indicator variable s_{t-1} takes the value of unity when $\varepsilon_{t-1} < 0$ and 0 otherwise. This process nests the standard GARCH(1,1) model when $\alpha_2 = 0$.

Following BPT (2001), standard GARCH style models are augmented by the inclusion of RV[7].

[6] Forecasts using the exponentially weighted moving average approach popularised by RiskMetrics was also considered intially. Forecasts from this model however were found to be redundant in the presence of the other models considered.

[7] While BPT (2001) also extend the GJR model to include the VIX index, this is not relevant to the current study.

The most general specification of a GARCH process including RV is given by,

$$\begin{aligned} r_t &= \mu + \varepsilon_t \qquad \varepsilon_t = \sqrt{h_t} z_t \qquad z_t \sim N(0,1) \\ h_t &= h_{1t} + h_{2t} \\ h_{1t} &= \alpha_0 + \beta h_{t-1} + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 \\ h_{2t} &= \gamma_1 h_{2t-1} + \gamma_2 RV_{t-1} \end{aligned} \tag{2}$$

and is defined as the GJR+RVG model. This allows for two components to contribute to volatility, with each component potentially exhibiting persistence. This specification nests various models, GJR+RV if $\gamma_1 = 0$, GARCH+RV if $\gamma_1 = \alpha_2 = 0$, GJR if $\gamma_1 = \gamma_2 = 0$ and GARCH if $\gamma_1 = \gamma_2 = \alpha_2 = 0$.

Parameter estimates for the GARCH and GJR models are similar to those commonly observed for GARCH models applied to various financial time series. The estimates reflect strong persistence in volatility and are qualitatively similar to those reported in BPT (2001)[8]. Furthermore, allowing for asymmetries in conditional volatility is important irrespective of the volatility process considered. In all cases the asymmetry parameter (α_2) is statistically significant and reduces the negative log-likelihood function significantly. Likelihood ratio (LR) tests of the validity of the symmetry restriction in *GJR*, *GJR+RV* are rejected (test statistics of 83.14 and 74.62 respectively exceed the χ_1^2 1% critical value of 6.63).

While not considered by BPT (2001), this study also proposes that an SV model may be used to generate forecasts. SV models differ from GARCH models in that conditional volatility is treated as a latent variable and not a deterministic function of lagged returns. The simplest SV models describes returns as

These models were used to extract information from VIX itself using forecasts based on historical data.

[8]As the models discussed in this section will be used to generate 2,460 recursive volatility forecasts (see Section 4) reporting parameter estimates is of little value. Here we will merely discuss the estimated model properties qualitatively. Parameter estimates for the recursive and an estimation based on the full sample are available on request.

$$r_t = \mu + \sigma_t u_t \qquad u_t \sim N(0,1) \tag{3}$$

where σ_t is the time t conditional standard deviation of r_t. The SV model treats σ_t as latent following its own stochastic path, the simplest being an AR(1) process,

$$\log(\sigma_t^2) = \alpha + \beta \log(\sigma_{t-1}^2) + w_t \qquad w_t \sim N(0, \sigma_w^2). \tag{4}$$

Similar to Koopman *et al.* (2004), this study extends a standard SV model to incorporate RV as an exogenous variable in the volatility equation. The standard SV process in equation (4) can be extended to incorporate RV in the following manner

$$\log(\sigma_t^2) = \alpha + \beta \log(\sigma_{t-1}^2) + \gamma(\log(RV_{t-1}) - E_{t-1}[\log(\sigma_{t-1}^2)]) + w_t. \tag{5}$$

Here, RV enters the volatility equation through the term $\log(RV_{t-1}) - E_{t-1}[\log(\sigma_{t-1}^2)]$. This form is chosen due to the high degree of correlation between RV and the latent volatility process and represents the incremental information contained in the RV series. It is noted that equation (5) nests the standard SV model as a special case by imposing the restriction $\gamma = 0$.

Numerous estimation techniques may be applied to the model in equations 3 and 4 or 5. In this instance the nonlinear filtering approach proposed by Clements, Hurn and White (2003) is employed. This approach is adopted as it easily accommodates exogenous variables in the state equation. The SV models appear to capture the same properties of the volatility process as the GARCH-type models. In both instances volatility is found to be a persistent process, and the inclusion of RV as an exogenous variable is important. A test of the restriction, $\gamma = 0$ is clearly rejected as the LR statistic is 156.22.

In addition to GARCH and SV approaches, it is possible to utilise estimates of RV to generate forecasts of future volatility. These forecasts can be generated by directly applying time series models, both short and long memory, to daily measures of RV, RV_t. In following ADBL (2003) and Koopman *et al.* (2004) ARMA(2,1) and ARFIMA(1,d,0) processes were utilised. In its most general form an ARFIMA(p,d,q) process may be represented as

$$A(L)\,(1-L)^{d}\left(x_t-\mu_{x_t}\right)=B(L)\,\varepsilon_t. \tag{6}$$

where $A(L)$ and $B(L)$ are coefficient polynomials of order p and q, and d is the degree of fractional integration. A general ARMA(p,q) process applied to x_t is defined under the restriction of $d = 0$. Here ARMA(2,1) and ARFIMA(1,d,0) models were estimated with $x_t = \sqrt{RV_t}$ and $x_t = \ln\left(\sqrt{RV_t}\right)$. These transformations were applied to reduce the skewness and kurtosis of the observed volatility data (ADBL, 2003). In the ARMA (2,1) case, parameter estimates reflect strong volatility persistence. Allowing for fractional integration in the ARFIMA(1,d,0) case reveals that volatility exhibits long memory properties.

The last approach considered here is one from the family of MIDAS forecasts. This methodology produces volatility forecasts directly from a weighted average of past information[9]. Following the notation introduced in Ghysels *et al.* (2006) a forecast is generated according to[10]

$$Q_{t+22,t}=c+\phi\sum_{k=0}^{k_{\max}}b\,(k,\theta)\,x_t+\varepsilon_t \tag{7}$$

where $Q_{t+22,t}$ represents the volatility forecast at time t for periods $t+1$ to $t+22$. Any variable containing information regarding the volatility process can be substituted for x_t. Examples used in Ghysels *et al.* (2006) were RV_t, squared daily return, absolute daily return, daily range and realized power variation. The maximum lag length $k_{\max}$ can be chosen rather liberally as the weight parameters $b\,(k,\theta)$ are tightly parameterised. Here the weights are determined by means of a beta density function and normalised such that $\sum b\,(k,\theta)=1$. A beta distribution function is fully specified by the 2×1 parameter vector θ.

Parameter estimation was achieved by nonlinear least squares (Ghysels *et al.*, 2004), minimising the sum of squared residuals in equation (7). If the methodology in Ghysels *et al.* (2006) were

[9] Formally Ghysels *et al.* (2006) forecast the quadratic variation of a price process, which is incidentally the same statistic that is proxied by the realized volatility measure.

[10] MIDAS models are more general as indicated by the notation here. They can deal with data being sampled at different frequencies and can also directly utilise intra-day data. These generalisations are not required here.

followed, $Q_{t+22,t}$ would only be sampled every 22 business days in order to avoid overlapping samples and the resulting residual correlation. As statistical inference, which would be complicated by such a residual structure is not of primary interest here, $Q_{t+22,t}$ is sampled daily[11]. All of the candidate x_t variables mentioned above, have been used with $k_{\max}$ of either 30 or 50. While all of these forecasts are highly correlated, the volatility forecast using absolute daily returns and $k_{\max} = 50$ is most highly correlated with VIX_t.

For the purposes of this study, no stance was taken in relation to the superiority of any forecasting model. Each model was treated as a potentially useful tool for generating volatility forecasts and were chosen so that the models are drawn from a range of model classes. The following section will describe how these models were used in the context of addressing our research question.

4 Methodology

The focus of this paper to test whether the VIX contains information incremental to that captured by MBF. Therefore it was necessary to decompose VIX_t, the realisation of the VIX observed at time t. The end product of this decomposition were two components of VIX_t, VIX_t^{MBF}, information in VIX that is captured by MBF and VIX_t^*, information in VIX not captured by MBF. VIX_t^* was constructed to be orthogonal to VIX_t^{MBF}. To generate VIX_t^{MBF} a linear projection of VIX_t into the space spanned by the MBF was used. In so far as this linearity assumption is restrictive, this methodology is biased toward rejecting the notion that VIX contains no incremental information. The relevant methodology is now discussed.

All MBF (at time t) of average volatility over the subsequent 22 trading days were collected in one vector, $\boldsymbol{\omega}_t$. These forecasts were based on the models discussed in Section 3 and generated on the basis of rolling window parameter estimates using 1,000 observations. The end of each window was the last observation before the 22 day forecast period[12]. The inclusion of RV into some of the

[11] The main motivation for this is that in later sections, when rolling forecasts are estimated, the sample size would be greatly reduced, resulting in unstable weighting functions.

[12] Descriptive statistics of these forecasts are presented at the beginning of Section 5.

volatility models discussed in Section 3 is problematic for the generation of multi-period forecasts as these require forecasts for RV itself. In order to avoid building a bi-variate forecasting model we follow BPT (2001) in postulating a linear relationship between RV_{t+k} and the conditional volatility forecast, σ^2_{t+k}, generated by the model under consideration[13].

Based on these forecasts, VIX_t was decomposed into

$$VIX_t = VIX_t^{MBF} + VIX_t^*,$$

which was required to ensure orthogonality between VIX_t^* and $\boldsymbol{\omega}_t$. A linear projection was used to map VIX_t into $\boldsymbol{\omega}_t$ with $VIX_t^{MBF} = \mathbf{Q}\ VIX_t$, where $\mathbf{Q}$ is the projection matrix of the stacked volatility forecasts in $\boldsymbol{\omega}_t$. VIX_t^* is then given by $VIX_t - VIX_t^{MBF}$ which is by definition orthogonal to the elements in $\boldsymbol{\omega}_t$. In practical terms this is equivalent to a linear regression

$$VIX_t = \gamma_0 + \boldsymbol{\gamma}_1 \boldsymbol{\omega}_t + \varepsilon_t \tag{8}$$

with $VIX_t^{MBF} = \widehat{\gamma}_0 + \widehat{\boldsymbol{\gamma}}_1 \boldsymbol{\omega}_t$ and $VIX_t^* = \widehat{\varepsilon}_t$.

If the VIX merely encompassed all of the information in the MBFs we would expect VIX_t^* not to contain any incremental information and to be orthogonal to future volatility (RV). In this instance, it would be expected that VIX_t^{MBF} forecasts future volatility equally as well as VIX_t.

A number of strategies can be used to implement the required orthogonality test between VIX_t^* and $\mathbf{RV}_{t+22}$, a generic vector containing information about future realisations of RV. The exact composition of $\mathbf{RV}_{t+22}$ will be discussed after the testing strategies have been introduced[14].

Estimation of

$$VIX_t^* = \boldsymbol{\beta}\ \mathbf{RV}_{t+22} + \upsilon_t \tag{9}$$

[13] See BPT (2001, p 14) for details of this procedure.

[14] Different versions of this vector will be introduced, but in general it will contain information about realised volatility up to date $t + 22$.

and a subsequent F-test, testing the null hypothesis that all elements in $\boldsymbol{\beta}$ are zero is required[15]. This is necessary as $\boldsymbol{\beta} = \mathbf{0}$ implies no relationship between VIX_t^* and the elements in $\mathbf{RV}_{t+22}$. As multi-step ahead forecasts were being considered, the residuals may be autocorrelated and an appropriate adjustment to the test statistic required (Harvey and Newbold, 2000).

The second testing strategy is related to the above approach, recognising that

$$\boldsymbol{\beta} = \left(E\left(\mathbf{RV}'_{t+22}\mathbf{RV}_{t+22}\right)\right)^{-1} E\left(\mathbf{RV}'_{t+22}VIX_t^*\right)$$

where $\boldsymbol{\beta}$ will only collapse to zero if $E\left(\mathbf{RV}'_{t+22}VIX_t^*\right) = 0$. The corresponding sample estimate, $\left(\mathbf{RV}'_{t+22}VIX^*\right)/n$, can be used to test the null hypothesis using the principle of Hotelling's generalised t-test (Hotelling, 1931). Again, an adjustment to the variance estimator of the test statistic, due to the use of overlapping observations is required. For this purpose the version of Hotelling's test as described in Harvey and Newbold (2000) was applied[16]. These tests have been established mainly in the context of forecast encompassing where their robustness to non-normal residuals has been investigated (Diebold and Mariano, 1995, Harvey, Leybourne and Newbold, 1997, 1998). While these test statistics have proven robust to excess kurtosis in residuals, these results have been established for short forecast horizons only (Harvey and Newbold, 2000). Therefore these current results should be interpreted with caution and their robustness checked by an alternative testing strategy.

The generalised method of moments (GMM) framework allows for a direct test of orthogonality conditions. A GMM estimate of $\boldsymbol{\gamma} = (\gamma_0, \boldsymbol{\gamma}'_1)'$ in (8) minimises $V = \mathbf{M}'\mathbf{H}\mathbf{M}$, where $\mathbf{M} = T^{-1}\left(\varepsilon_t(\boldsymbol{\gamma})'\mathbf{Z}_t\right)$ is a $k \times 1$ vector of moment conditions, $\mathbf{H}$ is a $k \times k$ weighting matrix and $\mathbf{Z}_t$ is a vector of instruments. In order to minimise coefficient variances, $\mathbf{H}$ was chosen to be the variance-covariance matrix of the k moment conditions in $\mathbf{M}$, where allowance was made for residual correlation (see Hamilton, 1994). Whenever $k > \dim(\boldsymbol{\gamma})$, the test for overidentifying

[15] Note that this regression is merely a tool to test a hypothesis and is not meant to imply that future realised volatility "cause" the VIX^*.

[16] The modified F and Hotelling-tests have $F_{\dim(\boldsymbol{\beta}),T-\dim(\boldsymbol{\beta})}$ distributions under the null hypothesis.

restrictions $J = T\mathbf{M}'\mathbf{H}\mathbf{M}$, is $\chi^2(k - \dim(\boldsymbol{\gamma}))$ distributed under the null hypothesis that the residuals in equation (8) are uncorrelated with elements in $\mathbf{Z}_t$. $\mathbf{Z}_t$ was designed to include MBF in $\boldsymbol{\omega}_t$, along with information regarding future realisations of volatility $\mathbf{RV}_{t+22}$, $\mathbf{Z}_t = \left(\boldsymbol{\omega}'_t, \mathbf{RV}'_{t+22}\right)'$. The J test was used to test the null hypotheses that VIX^*_t is orthogonal to all elements in $\mathbf{Z}_t$. This is equivalent to VIX_t not containing any incremental information relevant to the future realisations of volatility.

Finally it was necessary to discuss the composition of the vectors $\boldsymbol{\omega}_t$ and $\mathbf{RV}_{t+22}$. Each of the MBF discussed in Section 3 could be included in $\boldsymbol{\omega}_t$. However, as these elements are measuring the same quantity, they are highly colinear and therefore not all forecasts should be included in $\boldsymbol{\omega}_t$. A general-to-specific strategy within the context of a regression (8) eliminated the elements with the largest p-values leaving seven significant elements. They were the GARCH, GJR+RVG, SV, SV+RV, ARMA, ARFIMA and the MIDAS forecasts[17]. For the remainder of this paper, forecasts of S&P500 volatility over 22 business days, formed at time t, will be labeled GAR_t, GAR^+_t, SV_t, SV^+_t, AR_t, ARF_t and MAR_t respectively.

Before finalising the definition of $\boldsymbol{\omega}_t$ one additional issue must be addressed. Apart from the market's expectation of future volatility it has been shown that implied volatilities reflect a volatility risk-premium. As such, implied volatilities in general and the VIX in particular may be positively biased estimates of the market's expectation of future volatility (Chernov, 2002). This justifies the inclusion of the constant term γ_0 in equation (8). Furthermore, Chernov suggested the risk premium may be time-varying and correlated with the current level of volatility. To ensure the current results are robust to a potentially time-varying risk premium, a proxy for the current level of volatility, RV_t is included in $\boldsymbol{\omega}_t$:

$$\boldsymbol{\omega}_t = \left(GAR_t \; GAR^+_t \; SV_t \; SV^+_t \; AR_t \; ARF_t \; MAR_t \; RV_t\right).$$

[17] An alternative method to choosing elements is to select the principal components from the combination of the forecasts. Unpublished results show that the conclusions from this research remain unaltered if this route is followed.

On first sight one may also argue that the inclusion of RV_t into the information set can be justified without reference to the volatility risk premium, as it is available at time t[18]. In comparison to volatility forecasts, which all exhibit an element of smoothing, one may argue that RV_t contains unique information. However, as RV_t is included in a number of MBF its inclusion of RV_t in $\boldsymbol{\omega}_t$ should be superfluous from a forecasting perspective[19]. Therefore, the presence of a volatility risk premium appears to be the only justification for its inclusion.

The vector $\mathbf{RV}_{t+22}$ captures information regarding actual realisations of volatility during the 22 trading days following time t. While the VIX is designed to reflect volatility for the next 22 trading days, it is reasonable to include RV over shorter horizons to test whether VIX contains any incremental information with respect to shorter horizons. Let $\mathbf{RV}_{t+22}$ be defined as follows: $\mathbf{RV}_{t+22} = \{\overline{RV}_{t+1}, \overline{RV}_{t+5}, \overline{RV}_{t+10}, \overline{RV}_{t+15}, \overline{RV}_{t+22}\}$ where $\overline{RV}_{t+j}$ is the average RV in the days $t+1$ to $t+j$. Given that volatility is known to be a persistent process, forecasting not only the future level of volatility but changes in the level of volatility is of natural interest. To address the question of whether the VIX contains any incremental information beyond that contained in MBF with respect to the change in volatility, changes in the level of volatility from its current level are included as instruments. The inclusion of changes in RV is justified by one further consideration. The parameters in $\boldsymbol{\gamma}_1$ invariably sum to a number close to 1. This indicates that most of the level information in terms of volatility is captured by VIX_t^{MBF} and that VIX_t^* may contain useful extra information about changes in future volatility that cannot be anticipated from historical data used in MBF. In this spirit it is useful to investigate whether VIX_t^* is correlated with volatility changes. In addition to the above definition of $\mathbf{RV}_{t+22}$ the following alternative vectors incorporating information about future RV were used: $\mathbf{RV}_{t+22} = \{d\overline{RV}_{t+1}, d\overline{RV}_{t+5}, d\overline{RV}_{t+10}, d\overline{RV}_{t+15}, d\overline{RV}_{t+22}\}$ where $d\overline{RV}_{t+j} = \overline{RV}_{t+j} - RV_t$. Tests will also be conducted with constrained versions of $\mathbf{RV}_{t+22}$ following the finding of Fleming (1998) that IV is more strongly correlated with

[18] This is certainly true once the market closes in the afternoon.

[19] Forecasting volatility for days $t+1$ to $t+22$.

short-term future volatility.

5 Empirical results

Before addressing the research question a number of empirical features of the volatility forecasts contained in $\boldsymbol{\omega}_t$ will be described, emphasising their relation to $\overline{RV}_{t+22}$ being the proxy for volatility which is to be forecast. Table 1 will also provide the first insights into the relation between MBF and the VIX.

[Insert Table 1 here]

Not surprisingly, given the well known features of share market volatility, the realised average 22 day volatility is positively skewed and kurtotic. All volatility forecasts also exhibit these features with MBF having broadly similar means and medians. The correlation between MBF and $\overline{RV}_{t+22}$ gives some indication of the predictive ability of the different MBF. The correlations range between 0.6010 for the standard GARCH model and 0.7035 for the MIDAS forecast. These results concur with recent research which demonstrate that MIDAS type forecasting models (Ghysels *et al.,* 2006) generate superior forecasts.

Striking is the difference in the level of actual realised volatility and the VIX. Two potential reasons for this have been proposed. Equity markets unlike foreign exchange markets, have limited trading hours. This leads to RV being calculated on the basis of intraday returns covering 6.5 hours. The VIX however, is a proxy for volatility over the full calender day. To address this mismatch the overnight return was included as the first intra-day return when RV was calcuated and therefore cannot account for this difference[20]. As discussed earlier, the potential existence of a (time-varying) volatility risk premium may explain a mismatch between the actual RV and the VIX. Chernov (2002) demonstrates that a risk premium can drive a wedge between the VIX and the volatility expectation over the life of an option. He argues this usually leads to IV exceeding

[20] Some of this difference will also be due to different number of trading and calender days in a year, see Poon and Granger (2003) who suggest to divide the VIX by $\sqrt{365/252}$ to account for this difference. The VIX used here is appropriately rescaled.

the market's volatility expectation, which is consistent with the results presented here.

All forecasts are highly correlated with the current value of the VIX. Indeed the correlation is uniformly higher than that between the forecasts and their target. This correlation is not surprising and merely an indication of the VIX being largely based on the same information set as model based volatility forecasts.

The results of the modified F- and Hotelling-tests, testing the null hypotheseis that the VIX does not contain incremental information to MBF, are shown in Table 2 along with the parameter estimates for $\boldsymbol{\beta}$ in specification (9)[21].

The Table is divided into two panels which differ in the composition of $\mathbf{RV}_{t+22}$. The first line in the top panel refers to results with $\mathbf{RV}_{t+22} = \{\overline{RV}_{t+1}, \overline{RV}_{t+5}, \overline{RV}_{t+10}, \overline{RV}_{t+15}, \overline{RV}_{t+22}\}$. While the relation between $\overline{RV}_{t+1}$ (and $\overline{RV}_{t+5}$) and VIX_t^* appears to be marginally significant, the F- and Hotelling tests indicate that the null hypotheses of VIX_t not containing any incremental information cannot be rejected. The latter result is reinforced even in the case when the elements of $\mathbf{RV}_{t+22}$ are constrained to only contain individual $\overline{RV}_{t+j}$ ($j = 1, 5, 22$), with $j = 22$ being the time horizon matching the VIX. F-, Hotelling and t-tests all indicate that VIX_t does not contain any significant information beyond that contained in MBF (lines 2 to 4 in the top panel of Table 2), even at the 22 day horizon.

The informational content of VIX_t with respect to changes in the level of volatility ($d\overline{RV}_{t+j}$) is considered in the lower panel of Table 2. The results are slightly different because it appears as if the VIX_t^* is marginally significantly related to short-term changes in the level of volatility ($j = 1, 5$). However the economic importance of this would seem to be limited with the VIX_t^* explaining less than 1% of the variation in $d\overline{RV}_{t+j}$ in all cases considered here.

[Insert Table 2 here]

As discussed earlier, the empirical properties of the modified F- and Hotelling tests for long

[21] The parameter estimates from equation (8) are not reported at this stage but it turns out that about 84% of the variation in VIX_t is explained by variation in the model based volatility forecasts.

overlaps (here 21 observations) and data exhibiting excess kurtosis are not well established. A kurtosis measure of 8 for the VIX_t^* time-series highlights that these results should be interpreted with caution and the application of the GMM based tests is necessary to evaluate the robustness of these results.

The GMM estimation results pertaining to Equation (8) and associated J statistics to address the research question at hand are contained in Table 3. For the results presented here, the weighting matrix $\mathbf{H}$ was calculated using the Andrews and Monahan algorithm with pre-whitened residuals and automatic bandwidth selection. The automatic bandwidth selection algorithm selects bandwidth parameters around 25 and 30 corresponding well with the 21 day overlap in the data. To check the robustness of the results they were replicated using the Newey-West estimator of $\mathbf{H}$ with bandwidths of 21, 40 and 50. The results remain qualitatively unchanged and hence are not shown here[22].

[Insert Table 3 here]

Recall the definition of $\boldsymbol{\omega}_t = \{GAR_t, GJR_t^+, SV_t, SV_t^+, AR_t, ARF_t, MAR_t, RV_t\}$ used in equation 8. Results in the top row of Panel A relate to the instrument set $z_t = \{c, \boldsymbol{\omega}_t\}$. This represents an exactly identified system and thus the parameter estimates are equivalent to the OLS estimate of the vector $\boldsymbol{\gamma}$ in Equation 8. While these results do not directly address the research question they represent a benchmark for subsequent models. Overall, these results indicate that VIX_t is significantly related to each element of $\boldsymbol{\omega}_t$ ($R^2 = 0.84$) with positive coefficients relating to the GARCH and GJR+RVG forecasts indicating these models capture the overall level of volatility. Indeed, the sum of the two coefficients equals approximately one. The remaining forecasts included in $\boldsymbol{\omega}_t$ are SV_t and $SV + RV_t$; $ARMA_t$ and $ARFIMA_t$ and MAR_t. The first pair are of the stochastic volatility type and the second pair are time-series models of realised volatility. The former (latter) of each pair were found to have a negative (positive) relationship with VIX_t. It is conjectured that

[22] The results are available from the authors upon request.

the inclusion of RV into the SV model and the inclusion of long-memory features into the ARMA model contributes to explaining variation in VIX_t. As the restrictions that the respective coefficients have the same magnitude but opposite signs can be rejected, this interpretation is tentative. It should also be noted that in this and the following cases, the sum of all coefficients of elements in $\boldsymbol{\omega}_t$ is approximately one, as it should be, given that all elements in $\boldsymbol{\omega}_t$ are forecasts of the same object.

Results in the second row of Panel A are based on instrument sets allowing the application of the test for over-identified restrictions, the $J-$test. As it was argued in Section 4, this entails testing the null hypothesis that VIX_t does not contain incremental information. The J-test's p-value of 0.2545 suggests that there is no evidence for the presence of any incremental information in the VIX_t in relation to the future levels of volatility. Furthermore, the results in the next row relating to changes in the level of future volatility lead to the same conclusion. These results support those obtained from applying the modified F- and Hotelling tests discussed earlier. As in the case of the modified F- and Hotelling tests, the GMM test is also applied to reduced instrument sets including individual $\overline{RV}_{t+j}$ and $d\overline{RV}_{t+j}$ elements. The p-values of the ensuing J-tests are reported in Panel B of Table 3. The only rejection of the null hypothesis consistent across both sets of tests, is that relating to $d\overline{RV}_{t+5}$. However, this rejection appears to be marginal statistically and economically as discussed earlier.

The results presented in this section may be summarised as follows. It appears as though the VIX does not contain any economically important information incremental to that contained in MBF. This is not to say that the VIX is not useful for forecasting purposes, as has been demonstrated in previous research (see Poon and Granger 2003). Indeed the interpretation adopted here is that the VIX is a forecast combining useful elements from a range of volatility forecasting models. As MBF capture features of volatility such as the long-run mean and speed of and mean reversion in volatility, a corollary is that the VIX reflects no more information than these characteristics captured in MBF. The current results would explain a number of previous research findings discussed in Section 1. As the VIX may be viewed as a combination forecast, it is not surprising that its inclusion into any particular econometric forecast model will improve the model's forecasting performance, as in Day and Lewis (1992), Lamoureux and Lastrapes (1993) and in BPT (2001). Given that an IV such as the VIX, combine a wide range of available information captured in different volatility forecasts, it is not surprising that IV is often found to be a superior forecast relative to any single model (see Poon and Granger 2003). Indeed tests of the incremental information in VIX relative to individual MBF provide overwhelming evidence that the VIX does in fact contain information incremental to that contained in individual MBF[23]. Thus the current set of results, in combination with these earlier results, such as Fleming (1998) provide an more complete picture of the nature of the information compounded into IV.

6 Concluding remarks

This paper has examined the informational content of the VIX index, specifically, whether the VIX offers any incremental information to that contained in model based forecasts. Whilst numerous authors have considered the informational content of the VIX (along with other IV measures), none have sought to address this particular question. Detection of such information in the VIX would indicate the ability of the option market to anticipate the evolution of volatility in a fundamentally different way compared to more traditional volatility models.

In order to answer the question posed here it was necessary to decompose the information contained in the VIX into that which is correlated with model based forecasts and information that is not, which accounts for around 16% of the variation in the VIX. It is the behaviour of this component that is central to this study. Overall, the empirical results presented in Section 5 show that, if a wide range of model based volatility forecasts are considered, S&P 500 option IV (VIX)

[23]These results are available from the authors upon request.

does not contain any information regarding future volatility not captured by these forecasts. This indicates that the S&P 500 options market cannot anticipate movements in volatility unanticipated by model based forecasts.

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	RV22	GAR	GJRRVG	SV	SVRV	ARMA	ARFIMA	MIDAS	VIX
mean	1.0749	1.2639	0.9519	0.9760	1.3585	0.8175	0.8641	0.9632	1.6603
median	0.8163	1.1976	0.7553	0.8161	0.8161	0.7203	0.7898	0.8680	1.4424
s.d	0.9275	0.9790	0.8647	0.6939	1.0221	0.5903	0.6105	0.6632	1.0605
skew	2.1056	1.9313	2.4870	1.3214	1.5272	1.8489	1.6068	1.1429	1.5101
kurt	8.3375	9.3352	11.478	4.6645	7.1759	8.4984	7.0345	4.7940	6.0446
corr(·,RV22)	1.0000	0.6010	0.6808	0.6214	0.6672	0.6830	0.6751	0.7035	0.6778
corr(·,VIX)	0.6778	0.8309	0.8587	0.7615	0.7935	0.8529	0.8539	0.8638	1.0000

Table 1: Descriptive statistics for average 22 ahead RV, various volatility forecasts and VIX.

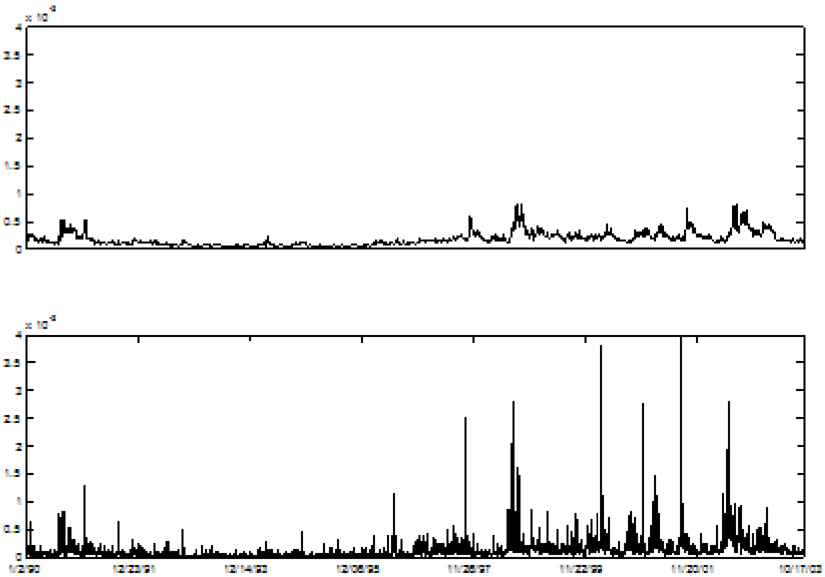

Figure 1: Daily VIX index (top panel) and daily S&P 500 index RV estimate (bottom panel).

$$VIX_t^* = \boldsymbol{\beta}\ \mathbf{RV}_{t+22} + \upsilon_t$$

Parameter estimates					Test statistics	
$j=1$	$j=5$	$j=10$	$j=15$	$j=22$	F	H
$\overline{RV}_{t+j}$						
-0.0164	0.0491	-0.0474	0.0315	0.0315	0.7848	0.8550
(-1.987)	(2.098)	(-1.227)	(0.499)	(0.499)	[0.561]	[0.510]
0.0206					0.3237	0.3310
(0.923)					[0.569]	[0.565]
	0.0410				0.7598	0.7828
	(1.356)				[0.383]	[0.376]
				0.0429	0.4851	0.4974
				(1.220)	[0.486]	[0.481]
$d\overline{RV}_{t+j}$						
-0.0134	0.0615	-0.0542	0.0602	-0.0236	0.7591	0.8258
(-1.740)	(2.281)	(-1.093)	(0.684)	(-0.261)	[0.579]	[0.531]
0.0197					3.8944	4.2506
(5.423)					[0.048]	[0.039]
	0.0344				2.9617	3.1767
	(5.391)				[0.085]	[0.075]
				0.0291	0.9327	0.9642
				(2.3051)	[0.334]	[0.326]

Table 2: OLS estimation results, $\widehat{\boldsymbol{\beta}}$, from equation (10). Six definitions of $\mathbf{RV}_{t+22}$. Non-empty cells in the Table correspond to the elements in $\mathbf{RV}_{t+22}$. Values in parentheses are t-statistics based on Hansen-Hodrick standard errors using a truncation parameter of 21. F-test (F) testing the null hypothesis that all parameters are equal to zero using an adjustment for autocorrelation of order 21. Hotelling's generalised t-test (H), testing the null hypothesis that the average cross products of VIX_t^* and the elements in $\mathbf{RV}_{t+22}$ are zero. Adjustments for autocorrelation of order 21 are applied according to Harvey and Newbold. p-values are shown in brackets.

$$VIX_t = \gamma_0 + \boldsymbol{\gamma}_1 \boldsymbol{\omega}_t + \varepsilon_t$$

Panel A

const	*GAR*	GJR^+	*SV*	SV^+	*AR*	*ARF*	*MAR*	*RV*	*J*
$z_t = \{c, \boldsymbol{\omega}_t\}$									
0.414	0.515	0.472	-1.064	0.235	-0.912	0.936	0.899	-0.058	NA
(8.56)	(7.43)	(3.44)	(-10.78)	(2.43)	(-2.02)	(1.84)	(3.53)	(-1.78)	
$z_t = \{c, \boldsymbol{\omega}_t, \overline{RV}_{t+1}, \overline{RV}_{t+5}, \overline{RV}_{t+10}, \overline{RV}_{t+15}, \overline{RV}_{t+22}\}$									
0.441	0.451	0.553	-0.931	0.134	-2.004	1.641	1.188	-0.084	0.2545
(8.52)	(4.97)	(5.10)	(-9.08)	(1.93)	(-3.79)	(3.16)	(5.19)	(-2.85)	(5)
$z_t = \{c, \boldsymbol{\omega}_t, d\overline{RV}_{t+1}, d\overline{RV}_{t+5}, d\overline{RV}_{t+10}, d\overline{RV}_{t+15}, d\overline{RV}_{t+22}\}$									
0.434	0.480	0.527	-0.970	0.149	-1.411	1.365	0.969	-0.098	0.2697
(8.64)	(7.46)	(4.92)	(-11.27)	(3.25)	(-2.85)	(2.75)	(4.36)	(-3.47)	(5)

Panel B

$z_t = \{c, \boldsymbol{\omega}_t, \overline{RV}_{t+j}\}$			$z_t = \{c, \boldsymbol{\omega}_t, d\overline{RV}_{t+j}\}$		
$j = 1$	$j = 5$	$j = 22$	$j = 1$	$j = 5$	$j = 22$
0.2360	0.0325	0.3010	0.2305	0.0689	0.2535

Table 3: Panel A: GMM estimates for the γ_0 and $\boldsymbol{\gamma_1}$ parameters in equation (8). The variable names represent the following forecasts included in $\boldsymbol{\omega}_t$: $GAR = GARCH$, $GJR^+ = GJR + RVG$, $SV = SV$, $SV^+ = SV + RV$, $AR = ARMA$, $ARF = ARFIMA$, $RV = RV$ and $MAR =$ MIDAS forecasts using absolute reurns and maximum lag of 50 days. z_t is the instrument vector for the GMM estimation. In parentheses, t-statistics for the coefficient estimates. J is the p-value for the test of overidentifying restrictions with degrees of freedom in parentheses. Significance tests were performed using the Andrews-Monahan weighting matrix with pre-whitening. Panel B: p-values for J-tests (1 d.o.f.) from GMM estimation of equation (9). z_t is the instrument vector for the GMM estimation.